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A Review of Fuzzy Computations

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Abstract:-In this paper a review of various models of fuzzy computations has been presented. Authors have analyzed more than 40 articles published during 1967 to 2012. The importance of fuzzy computations and the ever growing Industrial applications that implement some of the fuzzy computation models have also been stated. The paper aims to encourage additional research on some of the topics and concludes with suggestions for further research.

1. INTRODUCTION

Modern computer science can be said to have been initiated by the two outstanding papers [1] [2] written by Allen Turing in 1936. Turing developed in detail an abstract model for computation popularly known as the Turing machine (TM). This model formally defined what is meant by computation and the notion of an algorithm. Since the inception of TM a computation has been defined as a single transition on a TM while an algorithm to be a set of computations on a TM to solve a given problem. This abstract notion of TM has been accepted as what would be called a programmable computer. Further, Turing proved the existence of a universal TM that can be used to simulate any other TM which was algorithmic in nature. Subsequently all this led to the Church–Turing thesis which embodied the principle that if an algorithm can be performed on any piece of hardware then there exists an equivalent algorithm for a universal TM that would exactly perform the same task. Church–Turing thesis established the equivalence between the physical concept of what class of algorithm can be performed on some physical device and the rigorous mathematical concept of a universal TM. The wide acceptance of the thesis led to the foundation for the development of a rich theory of computer science. The classic book [3] that has continued to remain popular to date can be referred to for the exposition of this theory. The book richly explores formal models of computations such as finite automata, pushdown automata, TM and their variants; justly proving them to be the backbone of the field of computation. In 2009 Jeffrey Shallit brought out a book and called it a second course in formal languages and automata theory [4], a book the author states to have logically followed [3]. Computations that have been mainly characterized by the Church–Turing thesis are called “Crisp Computations.” Recent years have witnessed increasing interests in formal models of computations that go beyond the boundaries of classical computations/crisp computations. Further extrapolation of Church-Turing thesis considered other aspects such as interactions [11], real values [12], quantum universe [10], etc. cf.[13]. The research on fuzzy computability gained new strength, mainly because it was shown by Wiedermann [5, 14] that it is possible to solve the halting problem (more precisely, it is possible to accept r.e. sets and co-r.e. sets) in a class of fuzzy Turing machines, which got many researchers to pay attention to the formal models of fuzzy computations.

A precursor to fuzzy computability was the foundation of fuzzy sets which were introduced by Zadeh [6], since then they have found acceptance in many fields of sciences and in recent times in the industry too. Zadeh further extended the concept of fuzziness to algorithms [9]. The progress of algebraic theory of fuzzy automata and fuzzy formal languages can be seen by the amount of new research that has been reported over the years. Tatjana Petkovic’s work [8] is a good pointer to the same.

Formal models of computations lead to the concept of formal languages. A crisp formal language is defined to be the set of words accepted by a given formal model of computation. Alternately a crisp formal language can also be seen as a set of words over a given finite alphabet built according to specific rules or the grammar for that language. Formal languages mainly study the structure of the words and the relationship among languages. A fuzzy formal language is a formal language where each word has a degree of membership to the language.

In recent years study of fuzzy computations has been receiving steady but substantial attention due to its utility in diverse areas such as learning theory and learning systems[16], pattern recognition[17], fault tolerance decision making[18], genetics and biology[19,20], neural networks[21], Petri nets[23], coding theory[22] and natural language processing. Industrial applications in Robotics and Artificial Intelligence have been using fuzzy computations. Nondeterministic fuzzy automata have found application in nondeterministic fuzzy discrete event systems [24].

The paper reviews various formal fuzzy computation models besides fuzzy languages. Hence it is organized as follows: first, the research methodology used in the study is described; second, various articles related to fuzzy computation models and fuzzy languages have been considered and analyzed; finally the conclusions of the study are discussed.

Fuzzy computation involves study of the following:

1) Fuzzy finite automata and its variants, fuzzy regular expressions.
2) Fuzzy pushdown automata and its variants
3) Fuzzy Turing Machines and its variants
4) Fuzzy algorithms
5) Fuzzy grammars and fuzzy languages

2. RESEARCH METHODOLOGY

Research in Fuzzy Computations can be said to be confined to those journals which are devoted to Theoretical Computer Science besides fuzzy Mathematics etc. But research in the applications of fuzzy computations is difficult to confine to specific disciplines and hence they get published in various journals. Consequently following journals and online journal databases were searched.

1) ACM journals
2) Theoretical Computer Science
3) Elsevier
4) Fuzzy sets and systems
5) IEEE transactions
6) Science Direct

3) FORMAL MODELS OF FUZZY COMPUTATION:

A) Fuzzy finite automata and its variants, fuzzy regular expressions:

Wee in 1967 proposed the mathematical formulation of fuzzy automata for the first time [40, cf 24]. The basic idea as against the classical case was that a fuzzy automaton can switch from one state to another state to a certain possibility degree, and thus making it capable of capturing the uncertainty appearing in states or state transitions of a system. Cao, Ezawa's paper (2010)[24] has listed references of various variants of fuzzy automata proposed for various modeling situations. The paper mentions three types of fuzzy transition functions and also the use of algebraic structures such as lattices. The paper is concerned with fuzzy automata with nondeterminism and embedding fuzziness in nondeterminism; this idea differs from the notion of non deterministic finite automata put up earlier by some authors. The paper further proves equivalence between deterministic fuzzy automata, nondeterministic fuzzy automata and nondeterministic fuzzy automata with lamda moves. Choubey and Ravi have proposed finite automata with intuitionistic fuzzy (final) states (2009)[38]. The authors state that the finite automaton with intuitionistic fuzzy (final) states is more suitable for recognizing intuitionistic fuzzy regular language than earlier models.

B) Fuzzy Pushdown automata and its variants:

Hongyan Xing's paper (2007)[37] on fuzzy pushdown automata (FPDA) lists some of the earlier work carried out regarding FPDA. The similarity of fuzzy regular language to crisp case and the relationship between nondeterministic fuzzy finite automaton and fuzzy regular languages inspired Xing to study fuzzy pushdown automaton (FPDA) and fuzzy context-free languages (FCFLs). The paper, as a generalization of crisp pushdown automaton (PDA), studies and compares the behavior of an FPDA that is based on a more general lattice-ordered structure and discusses the character of multistack FPDA in accepting languages.

C) Fuzzy Turing Machines and its variants:

Yongming Li's paper (2008) [26] lists some of the earlier references related to the development of fuzzy TMs and its variants. He puts forth the all important question that if a universal fuzzy TM can be designed that could simulate any other fuzzy TM, stating the universal fuzzy TM to be very important from the point of view of fuzzy computations. He also criticizes that hardly any attention has been paid to the computability and complexity of fuzzy computation. Li's paper defines deterministic fuzzy TM (DFTM) and then studies the relationship between the DFTM and non deterministic fuzzy TM (NFTM). Firstly, he proves the DFTM and NFTM to be not equivalent as far as the power of recognizing fuzzy languages is concerned but to be equivalent in the power of deciding fuzzy languages. Second, the notions of fuzzy recursively enumerable (f.r.e.) languages and fuzzy recursive languages have been proposed and their stratified and grammatical description has been studied. Third, an attempt is made to study universal fuzzy computation in the frame of DFTMs and NFTMs and the results attained after comparing with ordinary TMs are given. It is also shown that the universal FTM exists only in the approximate sense. Lastly in order to efficiently study the computation realized in the setting of fuzzy sets, which is very important for the practical realization of fuzzy computation, the notions of fuzzy polynomial time-bounded computations (FPs) and nondeterministic fuzzy polynomial time-bounded computations (FNPs) are introduced, and their connections to *P* and *NP* are also characterized. Bedregal and Figueira have presented their work about Computability and TMs [41,42], the authors have studied the relationship between TM and classical recursion concepts such as computable functions, r.e. sets and universality. Further, they have stated necessary and sufficient conditions for a language to be r.e., by embedding it in a fuzzy language to be recognized by a fuzzy. By studying r.e. sets which the authors have stated to be a class of "harder" sets in terms of computability, the authors conclude that there is no Universal TM.

D) Fuzzy Algorithms:

Zadeh introduced notion of a fuzzy algorithm [9] as an ordered set of fuzzy instructions that, upon execution, yield an approximate solution to a given problem. Zadeh proves the necessity of the concept of Markoff algorithm than that of a TM to formalize the concept of a fuzzy algorithm because, as he states, an interesting aspect of Markoff algorithms relates to the fact that, as in the case of genetic algorithms the result of execution at each stage is a set of strings, rather than a single string. However, in the case of fuzzy Markoff algorithms, the set of strings is fuzzy, rather than crisp [27]. Santos and other researchers gave the formal description of the fuzzy algorithm by fuzzy variants of TMs, Markov algorithms and finite automata [26]. Since these beginnings, area of fuzzy algorithms has prospered well. Over the last four decades there has been a great deal of research in theory but more in applications of fuzzy algorithms. Many applications in the industry have been implementing fuzzy algorithms. There has been some research on the representation of fuzzy algorithms also. Mills and Harris have compared two apparently different approaches for representing fuzzy algorithms [28]: discrete and continuous. They state that traditionally, fuzzy algorithms have been implemented using a discrete approach where the fuzzy sets that form the rule base are defined as a set of discrete points, however, continuous fuzzy systems have recently gained in popularity, partly due to their links with certain classes of neural networks, but also because they generally require a smaller number of parameters and have a reduced computational cost. The review of fuzzy clustering algorithms [29] should act as a good pointer to the growth in the area of fuzzy algorithms.

E) Fuzzy grammars and fuzzy languages:

Fuzzy Grammars were proposed by Lee and Zadeh [30] who considered set of rewriting rules as fuzzy set. Since then an increasing amount of research has been done on integrating the concept of fuzziness, randomness or general weights with formal grammars and languages. Fuzzy grammars have been used in DNA sequence analysis [31]. The author has proposed fuzzy context-free grammars for the analysis of DNA sequences by using the Cocke-Younger-Kasami algorithm to estimate membership grades of a DNA sequence against the language of a fuzzy grammar etc. Fuzzy grammar has also been used in creating a texture segmentation prototype for industrial inspection applications [34]. Olgierd Unold has investigated the topic of a task of learning fuzzy context-free grammar from data for machine learning [35]. Fuzzy grammars have also been proposed to handle fuzzy algorithms [36]; the authors propose a procedure to transform the steps of a fuzzy algorithm into production rules of a fuzzy grammar and executing it using the dynamic of the grammar and adjusting its execution by means of the learning capability of the fuzzy formal languages. Authors state that the procedure is applied for executing and adjusting a fuzzy rule-based system.

If G is a fuzzy grammar then L (G) is the fuzzy language generated by the grammar G. Claudio Moraga introduced and discussed fuzzy Chomsky languages, fuzzy Petri Net languages and fuzzy Lindenmayer languages [32]. Moraga has also studied relationship between fuzzy context free languages and fuzzy push down automata [33]. In 2012 Ning, Wang, Zhang have introduced pragmatic functions of fuzzy language [39] and have proposed strategies for the translation of fuzzy languages.

4) CONCLUSION:

In this study the authors have conducted an extensive review of academic articles and have provided a comprehensive bibliography in the area of fuzzy Computations. Although the study cannot be claimed to be exhaustive and complete; authors believe that it should prove to be a useful resource for anyone who would be interested in fuzzy Computations research. Authors would like to extend present research in the area of fuzzy Finite Automata, fuzzy Turing Machines and fuzzy Languages. Authors also intend to work in the area of fuzzy Algorithms. Fuzzy Compilers is an interesting area for further research. Developing models of fuzzy computations that will help in creating algorithms for a fuzzy compiler remains an interesting area. Fuzzy context sensitive languages besides a compiler for natural languages remain a challenging task and a good area for further research.

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